Hello! - Petru Constantinescu, postdoc with Philippe Michel - Exercise Sessions are important! Students present their solutions, active participation encouvaged! - Learning process is active, not passive!
- Exam: written, modelled after the exercises. Topic of the course Let N:= {1,2,3,...} Goal: Understand multiplicative structure of N

P:= & R & N : R prime }

-> multiplicative building blocks of N Fundamental theorem of arcthmetic

For all new, 3! Vn: P -> N U EO ] such that  $V_n(p) = 0$ , for almost all  $p \in P$ and  $n = \prod_{p \in P} p^{v_n(p)}$ , i.e.  $n = p_1 \cdots p_n^{l_n}$ .

Goal: Understand the set P. Theorem (Euclid) 11P1 = 0. Proof: Suppose for contradiction P is finite, so P= Ep2,..., pr]. Let q:= 1+ p2...pr.
Then pi+q, ti. Therefore qep, controdiction. Let  $T(x) = \# \sum_{p \leq x} p \text{ prime}_{S}$ Exercise  $T(x) \geq \log \log x$ .

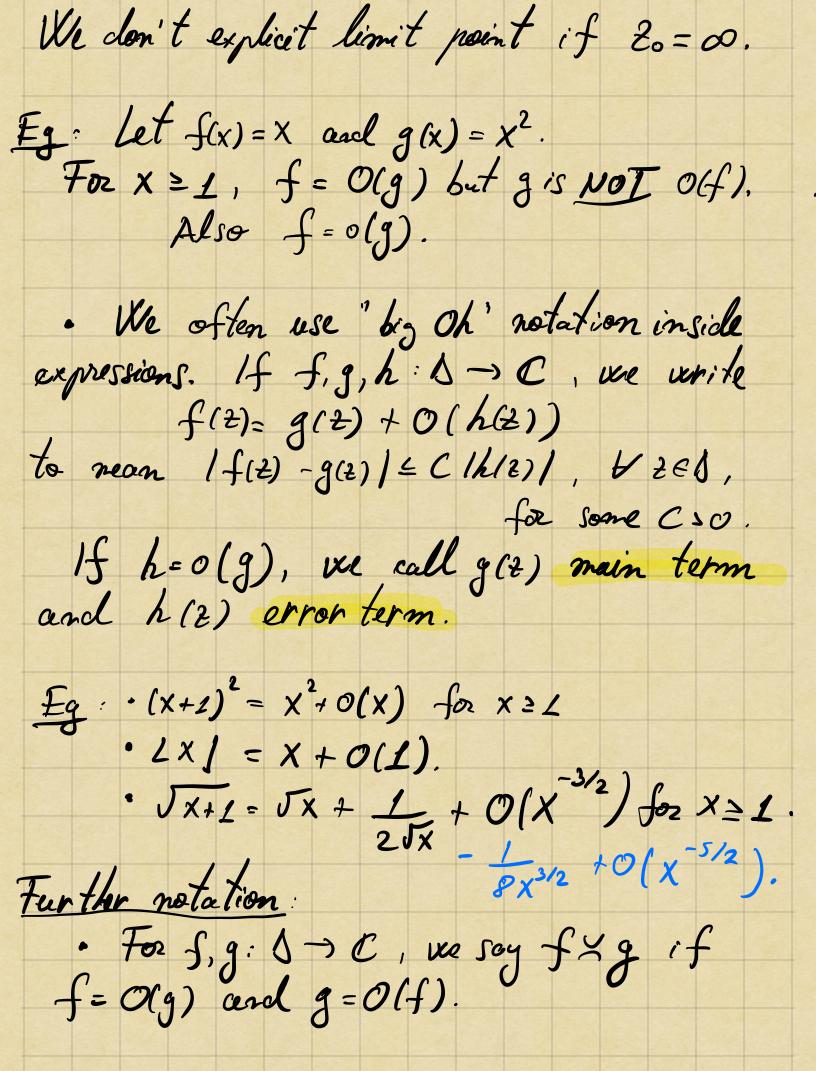
Refinements. · Gauss conjectured that  $\pi(x) \sim x$ ,
i.e. lim  $\pi(x) = 1$ .  $x \to \infty$  x/leyxThis is true, and is known as the Prine Number Theorem (we prove it in this course). It was proven in 1896 by Hadamard and de la Vallée Poussin (independently), more than 100 years often bauss much the conjecture.

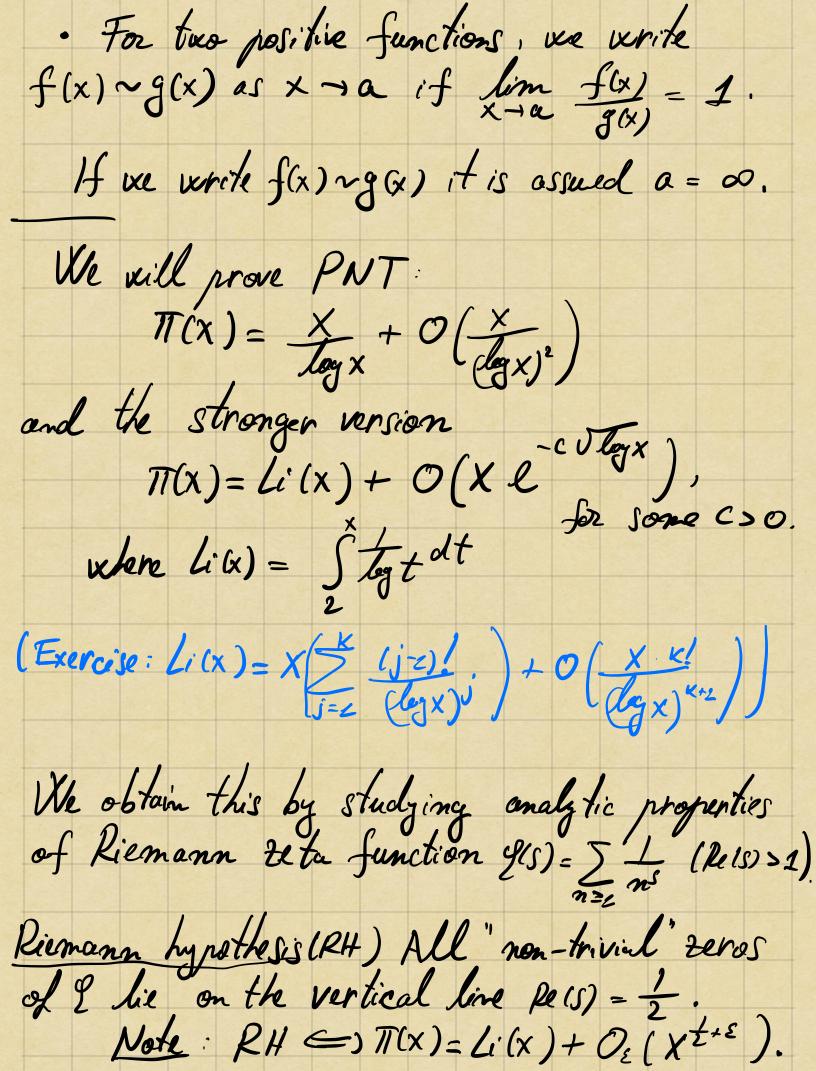
· Cl: Are there infinitely many primes with last cligit 3?

More generally, if (a,b) = 1, is

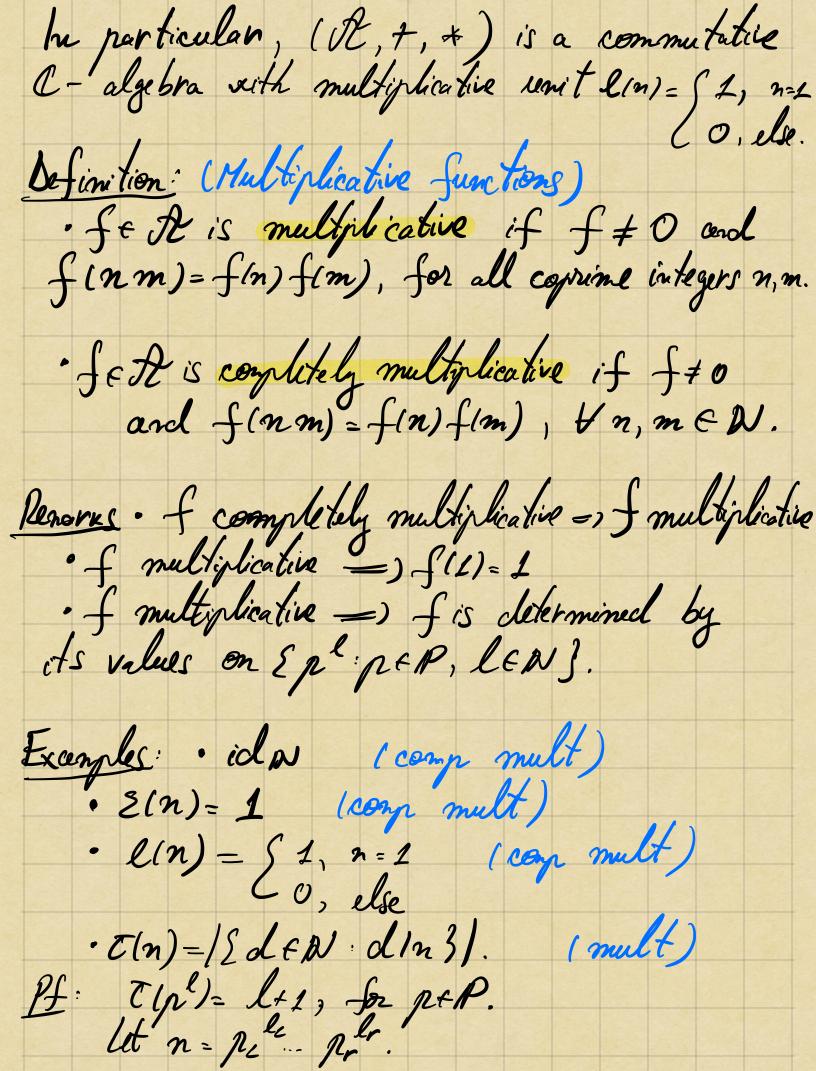
15 ant b: nEN3 DP 1= 00? Dirichlet: ges! Proportion of primes = 6 (moda) is fra).
This is Prime Number Theorem in Arithmetic Progressions. · 18 18 n+1: n+N3 0P1= 00? Wide open. Notation: · Let  $f,g:b \rightarrow C$  ( $b \subseteq C$ ). We say f=O(g) or f=Q(g) if there is a constant C>0 s.t. 1f(2)1 = c/g(2)1, 4200 • Let  $t \in D$  and  $f, g: D \setminus S \setminus S \setminus S \rightarrow C \cdot S \cdot f$ .

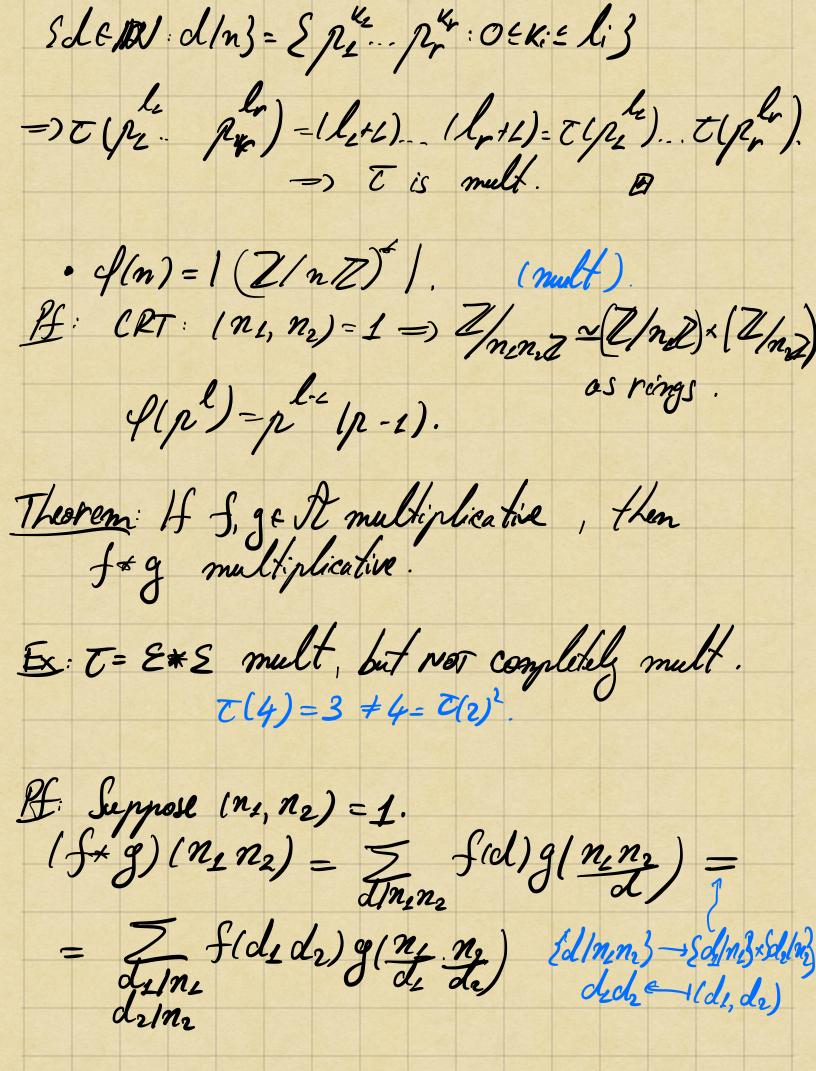
g does not vanish man  $t \in V$  that f = o(g)as  $t \rightarrow 2 \circ if$   $\lim_{t \rightarrow 2 \circ 2 \circ S(2)} f(2) = 0$ .





Arithmetic functions  bef: An arithmetic function is a function $f: N \to C$ . Denote $P = f \to N \to C f$ the set of all arithmetic functions.  bef (birichlet convolution). Let $f, g \in R$ .  Dirichlet convolution $f * g \in R$ is defined $(f * g) (n) = f(d) g(n)$ .	
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lemme: Hf,g,h&&	
· f * 9 = 9 * f (x is commutative)	
empe : $f,g,h \in \mathcal{U}$ • $f * g = g * f                           $	e)
$\frac{P_{roof}}{roof}$ : $(f \neq g)(n) = \sum_{n=ab} f(a) g(b) - \sum_{n=ab} f(b) g(a) - g(a)$	361
neab neab	4 %
(Ca) 11/2 - 5 1/12 5 f(a) = (1)	
(fag) = L)(n) = \( \sum_{n=06} h(b) \) \( \sum_{e} = cd \) (c) g(d) =	
= \( \sharp h(b) f(c) g(d) = (f*(g*h)) (n).	





= Z f(de) g(nx) Z f(de) g(nx) delm - (frg) (n2). (frg) (n2). Units in It Thorem fest unit => f(1) #0. If: "==" Suppose 3 gets st. fx g = l.

Then 1= e(1)= f(1)g(1)=> f(1) \$0. 'E' We inductively define get st. fx g= e.

Set g(1)=1

f(1) Suppose g(1), g(n-1) already defined. Set  $g(n) := -\frac{1}{5!1} \sum_{d \in I} f(d)g(n)$ . Then fry(n) = e(n), unew. (as evere) & Rmx: We denote the inverse of f by ft.

Proposition: If fett multiplicative, then so is fit. Prof: let get t multiplieatie defined by
g(p)= f-1(p'), peP, lew. = f\* f\*) (p)=e(p).

T f\* g = l, as both f\* g arel l are determined uniquely by their values on prine powers.

By uniqueness of inverse, g = f'. Example: let  $\mu := \mathcal{E}^{-1}$  Mobius Junction. Lemma:  $\mu(n) = S1$ , n = 1  $(C-1)^n, n = p_1...p_r$ distinct prines. 0, else.Note: 141=inclicator Set of Square-free numbers.

B: 141=1

